A NEW FINITE-DIFFERENCE TIME-DOMAIN FORMULATION EQUIVALENT TO THE TLM SYMMETRICAL CONDENSED NODE

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ABSTRACT

A new Finite-Difference Time-Domain (FD-TD) formulation is proposed. It is shown to be exactly equivalent to the symmetrical condensed node model used in the Transmission Line Matrix (TLM) Method. Due to a better field resolution and fulfillment of continuity conditions, the new FD-TD formulation or its TLM equivalent model have less dispersion and better accuracy than the traditional FD-TD method based on Yee's scheme.

1 INTRODUCTION

Recently, time domain solutions for field problems have received growing attention. Two currently employed techniques are the Transmission-Line Matrix (TLM) method[1] and the Finite-Difference Time-Domain (FD-TD) method formulated by Yee[2]. Both methods have been successfully applied to solve electromagnetic field problems in the time-domain [3]–[4]. In this paper, a new finite-difference time-domain formulation for Maxwell's equations is proposed and is shown to be exactly equivalent to the TLM condensed node algorithm in terms of field quantities.

2 A NEW FINITE-DIFFERENCE TIME-DOMAIN FORMULATION FOR MAXWELL'S EQUATIONS

Maxwell's equations in a stationary and sourceless medium in the time-domain can be expressed in a rectangular coordinate system. For instance, considering E_x, H_y, H_z, one has

$$\varepsilon\frac{\partial E_x}{\partial t}=\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z} \tag{1}$$

Following Yee's notation[2], we denote:

$(i,j,k)=(i\Delta x, j\Delta y, k\Delta z), F(i\Delta x, j\Delta y, k\Delta z, n\Delta t)={}_nF(i,j,k)$

where Δx, Δy, and Δz are the space increments, Δt is the time increment, and i,j,k,n are integers or integers$\pm\frac{1}{2}$.

Now, instead of defining the field components of **E** and **H** on the mesh like Yee[2] (See Fig.1a), we position them in the way depicted by Fig.1b. Thus, differencing (1), one can have

$$\varepsilon\frac{{}_{n+1}E_x(i,j,k)-{}_nE_x(i,j,k)}{\Delta t}=\frac{{}_{n+\frac{1}{2}}H_z(i,j+\frac{1}{2},k)-{}_{n+\frac{1}{2}}H_z(i,j-\frac{1}{2},k)}{\Delta y}-\frac{{}_{n+\frac{1}{2}}H_y(i,j,k+\frac{1}{2})-{}_{n+\frac{1}{2}}H_y(i,j,k-\frac{1}{2})}{\Delta z} \tag{2}$$

The remaining finite difference equations related to the other five field equations can be similarly constructed.

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3 VOLTAGE AND CURRENT RELATIONS IN THE SYMMETRICAL CONDENSED NODE OF TLM AND ITS EQUIVALENCE TO THE NEW FINITE DIFFERENCE APPROACH

Without loss of generality, consider a symmetrical condensed node without inductive, capacitive and loss stubs (Fig.2) , as proposed by Johns [5]. Note that the scattered voltages are related to the incident voltages through the scattering matrix [S] at node (i,j,k), and the total voltages at the nodes are also related to the incident voltages, given in [5]. Then it is not difficult to show that:

$$2\,C\frac{{}_{n+1}V_x(i,j,k) - {}_n V_x(i,j,k)}{\Delta t} = -\frac{{}_{n+\frac{1}{2}}I_{y12}(i,j+\frac{1}{2},k) - {}_{n+\frac{1}{2}} I_{y1}(i,j-\frac{1}{2},k)}{\Delta l} - \frac{{}_{n+\frac{1}{2}}I_{z9}(i,j,k+\frac{1}{2}) - {}_{n+\frac{1}{2}} I_{z2}(i,j,k-\frac{1}{2})}{\Delta l} \quad (3)$$

where ${}_nV_x(i,j,k)$ is the total voltage at node (i,j,k) and time $n\,\Delta t$. ${}_{n+\frac{1}{2}}I_{wp}(i,j\pm\frac{1}{2},k)$ or ${}_{n+\frac{1}{2}}I_{wp}(i,j,k\pm\frac{1}{2})$ is the current flowing in link line p along the w-direction at time $(n\pm\frac{1}{2})\Delta t$ and grid-point $(i,j\pm\frac{1}{2},k)$ or $(i,j,k\pm\frac{1}{2})$.$(w=y,z$ and $p=1,2,9,12)$.

If the voltages and currents defined above are associated with the appropriate field components as indicated in [5]:

$V_x \equiv E_x, I_z \equiv H_y, -I_y \equiv H_z, 2C \equiv \varepsilon, \Delta l = \Delta x = \Delta z$ at any time and grid points, then (3) is exactly the same as (2).

The remaining five equations can be derived in a similar manner by assuming $V_y \equiv E_y$, $V_z \equiv E_z$ and $\mu \equiv 2\,L$ in addition to the above equations, where L and C are the inductance and capacitance per unit length of the link lines. Thus, the symmetrical condensed node TLM model is numerically equivalent to the new FD-TD formulation for Maxwell's equations. One can easily verify that the same conclusion will be reached by following a similar procedure for a condensed node with stubs.

4 NUMERICAL RESULTS

Fig.3 shows the numerical results for the normalized cutoff frequency of the finned waveguide. It can be seen that the solution of the new FD-TD formulation or TLM algorithm converges faster than the traditional FD-TD method based on Yee's scheme as the number of iterations is increased. However, it was found that if the same accuracy is required, both FD-TD methods take almost the same total CPU time since each iteration of the new FD-TD approach needs slightly more CPU time than that of the traditional FD-TD method. Nevertheless, the new FD-TD approach gives better field resolution simply because in the new FD-TD formulation, more field components, including both tangential electric and magnetic field components at points in between the nodes, are computed. Also, the field components defined are not separated in space in the new FD-TD formulation, resulting in less frequency dispersion of the numerical scheme.

5 CONCLUSION

The proposed FD-TD formulation results in a kind of 'condensed' model where both electric and magnetic field components are defined at the center of a unit cell and at mid-points between adjacent nodes. As a result, it has better resolution and accuracy than Yee's scheme for solving electromagnetic problems. In addition, it is shown that the condensed node TLM algorithm can be exactly translated into the new finite-difference time-domain formulation.

References

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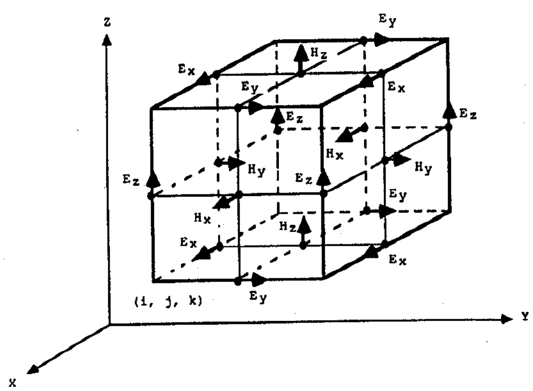


Fig.1(a) Positions of the field components about a unit cell of the Yee lattice

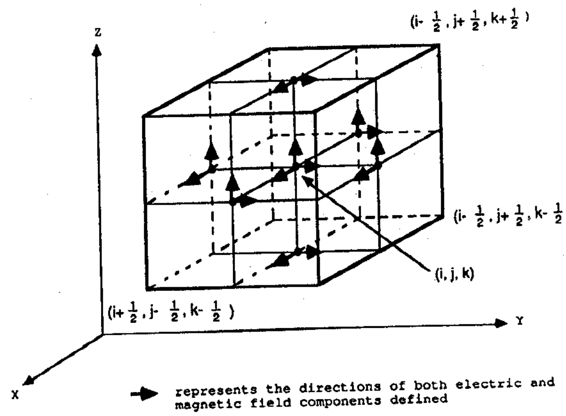
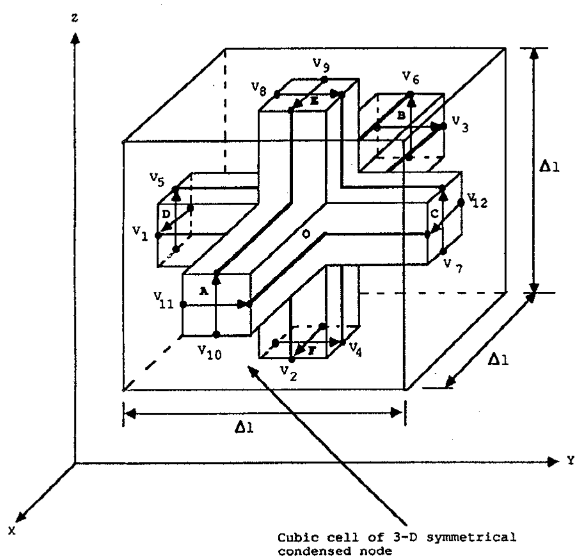


Fig.1(b) Positions of the field components about a unit cell of the new finite-difference time-domain formulation



Positions:

O: (i, j, k)

A: $(i+\frac{1}{2}, j, k)$ B$(i-\frac{1}{2}, j, k)$

C: $(i, j+\frac{1}{2}, k)$ D$(i, j-\frac{1}{2}, k)$

E: $(i, j, k+\frac{1}{2})$ F$(i, j, k-\frac{1}{2})$

Fig.3 The symmetrical condensed node

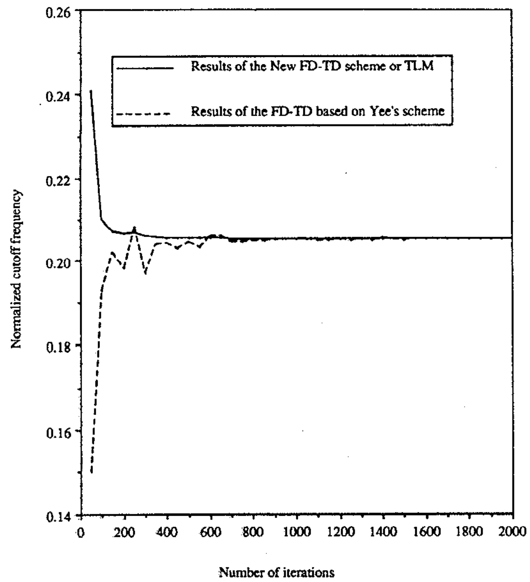


Fig.4 Normalized cutoff frequency in the rectangular finned waveguige obtained with increased number of iterations